The game of Cops and Attacking Robbers

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February 25, 2026

PIMS

It all starts with a robbery

The scenario

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The cops want to catch the robber. The robber wants to avoid capture.

Mathematical framework

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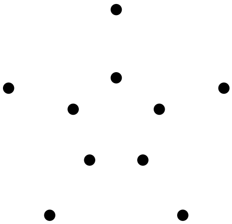

Figure: Vertices of a graph.

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We will play on a combinatorial object called a graph!

A graph G is built with a set of vertices and set of edges drawn between vertices.

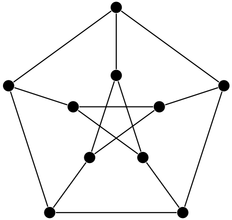

Figure: Drawing edges.

The game of Cops and Robber played on graphs

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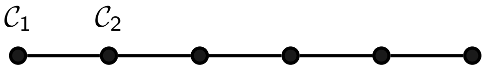

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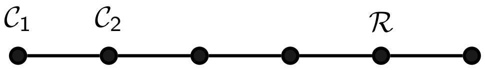
2. A single robber, $\mathcal{R}$.



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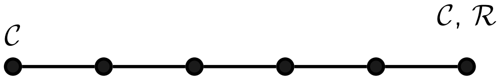
2. The robber then places themselves on a vertex;
3. Both sides play with perfect information;
4. During their turn, each cop makes a move by sliding across one edge or passing;
5. Likewise for the robber;
6. The cops win if one of them occupies the same vertex as the robber.



Optimization

Goal: We want to minimize the number of cops used to capture the robber.

Definition 1

In general, G is k-**copwin** if k cops have a winning strategy on G. The **copnumber** of a graph is defined as follows:

$$c(G) = \min\{k \mid \text{G is } k\text{-copwin}\}.$$

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We can't win with one cop on the graph below!

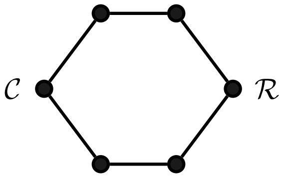

Figure: A cycle on 5 vertices.

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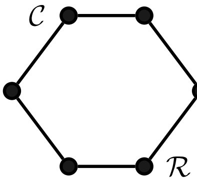

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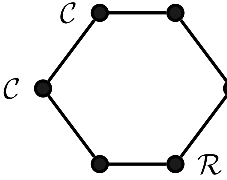
However, the cops win with two!



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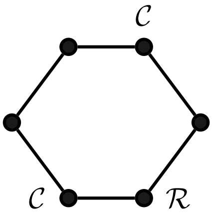

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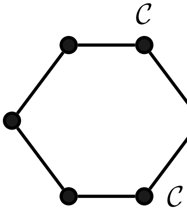

Figure: A cycle on 5 vertices.

This graph has copnumber 2.

What do we know?

- 1-copwin graphs are characterized (Nowakowski and Winkler, Quilliot);
- k-copwin graphs are characterized (Clarke and MacGillivray);
- We have found a d such that $c(G) \leqslant d$ for several classes of graphs such as:
  - Planar graphs;
  - Graphs with genus g;
  - Generalized Petersen graphs;
  - etc....

Research directions

- Can we bound the copnumber from above using other graph theoretic properties?
- Can we minimize the number of turns required to capture the robber?
- What if we impede the cops?

Making life difficult for the cops

There exist numerous variations that impose some constraint on the cops:

- Restricting the cops' visibility.
- Speeding up the robber.
- Giving the robber the ability to defend themself.

The game of Cops and Attacking Robber

Introduced by Bonato et. al (2013).

We will follow the same rules as the original game of Cops and Robber with one modification.

The attacking robber: If the robber is adjacent to a cop at the beginning of his turn, he can **attack** this cop by moving on that cop's vertex and eliminate this cop from play.

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Definition 1

The **cc-number** of G, denoted $cc(G)$, is the minimum number of cops for which there exists a winning strategy in the game of Cops and Attacking Robber.

Question: How does the cc-number of G relate to $c(G)$?

Answer

For all graphs G, we have:

$$c(G) \leqslant cc(G) \leqslant 2c(G).$$

We can win with $c(G)$ **pairs** of cops.

Observation

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The cops can buddy-up to keep each safe.

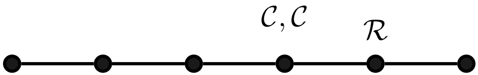

Figure: Two cops are buddied-up.

Research question

Main question: For each $k \in \{1, 2, 3, \ldots\} = \mathbb{N}$, can we find a graph such that $c(G) = k$ and $cc(G) = 2k$?

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Answer: Yes!

Theorem 1 (Brendle, Clow, LM, Ueckerdt, 2026+)

For all $k \in \mathbb{N}$, there exists a graph G such that $c(G) = k$ and $cc(G) = 2c(G)$.